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Quantum Zeno effect in the measurement problem¹

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Abstract

Critically analyzing the so-called quantum Zeno effect in the measurement problem, we show that observation of this effect does not necessarily mean experimental evidence for the naive notion of wave-function collapse by measurement (the simple projection rule). We also examine what kind of limitation the uncertainty relation and others impose on the observation of the quantum Zeno effect.

1 Introduction

The quantum Zeno paradox, named after the famous Greek philosopher Zeno, states that an unstable quantum system becomes stable (i.e. never decays) in the limit of infinitely frequent measurements. However, we cannot observe this limit, practically and in principle, as will be seen later on the basis of the uncertainty relations. Physically, its milder version, i.e. the quantum Zeno effect (QZE) can be observed. On the other hand, one may consider as if observation of this kind only of phenomenon were a clear-cut support for the naive notion of wave-function collapse (WFC) (the simple projection) onto an observed state. The purpose of this paper is to contrast this kind of misunderstanding by analyzing the mechanism of QZE and to discuss the important role of the uncertainty relations in observation of QZE.

A quantum system that is initially prepared in an eigenstate of the unperturbed Hamiltonian undergoes a temporal evolution that can be roughly divided into three steps [1]-[6]: A Gaussian-like behavior at short times, a Breit-Wigner exponential decay at intermediate times, and a power law at long times.

The first idea of the QZE was introduced under the assumption that the Gaussian short-time behavior can be observed in a quantum decay and only the naive WFC takes place in quantum measurements [7] [8]. In this context, the QZE is closely connected to the WFC by measurement, so that we have to examine one of the central measurement problems: What is the wave-function collapse? The authors have formulated a reasonable theory of measurement without resorting to the naive WFC [9][10]. In this paper, however, we shall not discuss this problem (see refs). Rather, we shall explain the QZE along the line of thought of the naive WFC.

Usually, the Gaussian decay is very difficult to observe. For this reason, the QZE was not considered to be easily amenable to experimental test until Cook [11] proposed using atomic

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transitions in two-level atoms. On the basis of this idea, Itano and his group [12] recently carried out an interesting experiment, claiming that they experimentally justified the naive WFC by observing the QZE. As opposed to this, Prigogine and his group asserted, by theoretically deriving the same result only through a dynamical process without the help of the naive WFC, that this experiment did not necessarily support the naive WFC. This provoked an interesting debate [13].

The present authors and others proposed to use neutron spin-flip, instead of atomic transitions, in order to confirm and simplify (and generalize) Prigogine et al's theory [14]. Similar kind of experiments were proposed and recently performed by making use of photon polarization [15][16].

It is also important to note that Itano et al's experiment did not observe the state every time except the final one, and therefore was not exactly the same as Cook proposed. One of the main interests in this paper is, therefore, to find the reason why their experiment could give the same result as Cook's prediction, which was given by assuming the naive WFC (a simple projection) at every step.

2 Naive formulation of QZE

We first formulate the QZE along the line of thought of the naive WFC, and discuss it as an dynamical process in the next section.

For initial state u_a at $t = 0$ (an eigenstate of unperturbed Hamiltonian \hat{H}_0), the wave function dynamically changes as

$$\psi_t = \exp(-iE_a t - \frac{1}{2}F_a t^2)u_a + O(t) : F_a \equiv (u_a, \hat{H}_1^2 u_a) \quad (1)$$

at very short $t(> 0)$, provided that $(u_a, \hat{H}_1 u_a) = 0$.

According to the idea of the naive WFC, the system suffers such a sudden change as

$$\psi_t \implies \exp(-iE_a t - \frac{1}{2}F_a t^2)u_a \simeq (1 - \frac{1}{2}F_a t^2)u_a e^{-iE_a t} \quad (2)$$

for its wave function, or

$$\rho_t \implies \exp(-F_a t^2)\rho_{aa} \simeq (1 - F_a t^2)\rho_{aa} : \rho_{aa} \equiv |u_a \rangle \langle u_a| \quad (3)$$

for its density matrix, at very short t .

The probability of finding state u_a at very short $t(> 0)$ is given by

$$P_a(t) = (1 - F_a t^2) . \quad (4)$$

Therefore, the probability of finding the same state N times by repeated measurements of this kind in time intervals $(0, T/N), \dots, ((N-1)T/N, T)$ (note that $t = T/N$ for one step) during $(0, T = tN)$ is given by

$$P_a^N(T) = \left[1 - F_a \left(\frac{T}{N} \right)^2 \right]^N \xrightarrow{N \rightarrow \infty} 1 . \quad (5)$$

We propose to distinguish the QZE from the quantum Zeno paradox in the following way:

Quantum Zeno paradox:

$$\lim_{N \rightarrow \infty} P_a^N(T) = 1 \quad : \quad \text{only in the infinite } N \text{ limit,} \quad (6)$$

Quantum Zeno effect:

$$P_a^N(T) > P_a^{N'}(T) \quad , \quad \text{if } N > N' \quad : \quad \text{for finite } N \text{ and } N'. \quad (7)$$

Remember that we have simply formulated the quantum Zeno paradox and the QZE by making use of the Gaussian decay and the naive WFC.

3 Neutron spin-flip and discussion on QZE

In Cook's case, we observe only the temporal evolution of the type $\cos^2(\omega/2)$, so that we obtain

$$P_a^N(T) = \left(\cos^2 \frac{\pi}{2N} \right)^N \quad (8)$$

for the probability of finding the initial state u_a at time T after N -step measurements, if we choose T so as to give $\cos(\omega T/2) = 0$.

In the neutron spin-flip case, we can also formulate the theoretical procedure in a similar way as in Cook's case, if we use a polarized neutron beam along the z -axis and N magnetic fields with strength B along the x -axis as shown in Fig.1 (Case A), where $\omega = \mu B/\hbar$ (μ being the neutron magnetic moment). Therefore, we can describe the one-step measurement as

$$\begin{aligned} \rho(t) &= \rho_{aa} \cos^2 \frac{\omega t}{2} + \rho_{bb} \sin^2 \frac{\omega t}{2} \\ &\quad - i \rho_{ab} \cos \frac{\omega t}{2} \sin \frac{\omega t}{2} + \text{h.c.} \\ \implies \rho_{aa} \cos^2 \frac{\omega t}{2} &\quad (\text{naive WFC projection}) \end{aligned} \quad (9)$$

where $a = \uparrow$, $b = \downarrow$, $t = T/N$ (or $\omega t/2 = \pi/2N$), and then the final density matrix after N -step measurements becomes

$$\rho_A^N(T) = \left(\cos^2 \frac{\pi}{2N} \right)^N \rho_{aa} \quad , \quad (10)$$

and correspondingly, we can get the probability of finding the upward spin state at time T in the same form as (8) with $a = \uparrow$.

In this case, we can explicitly write down the whole density matrix in the channel representation before the final spin-detection in the following way:

$$\rho_A^N(T) = \begin{pmatrix} c^{2N} & & & & 0 \\ & s^2 c^{2N-2} & & & \\ & & s^2 c^{2N-4} & & \\ & & & \ddots & \\ 0 & & & & s^2 \end{pmatrix} ,$$

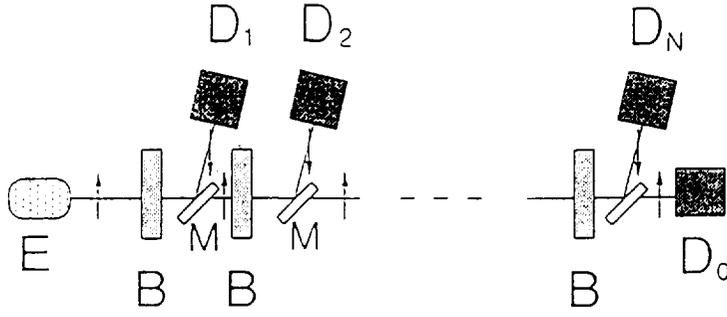


Figure 1: QZE in case A.



Figure 2: QZE in case B

where $c = \cos(\pi/2N)$ and $s = \sin(\pi/2N)$.

On the other hand, we know that a measurement process can be divided into two steps, the first being the spectral decomposition and the second the detection. Usually, spectral decomposition step is a sort of dynamical process that keeps coherence among the branch waves. In this case, the experimental procedure is illustrated in Fig. 2 (Case B). We can easily show that, through the N -step spectral decompositions, the density matrix of the system will dynamically change as

$$\rho_B^N(T) = \left(\cos^2 \frac{\pi}{2N} \right)^N \rho_{aa} + \text{other components} . \quad (11)$$

Therefore, we can explicitly write down the whole density matrix in the same channel representation before the final spin-detection as follows:

$$\rho_B^N(T) = \begin{pmatrix} c^{2N} & isc^{2N-1} & isc^{2N-2} & \dots & isc^N \\ -isc^{2N-1} & s^2c^{2N-2} & s^2c^{2N-3} & \dots & s^2c^{N-1} \\ -isc^{2N-2} & s^2c^{2N-3} & s^2c^{2N-4} & \dots & s^2c^{N-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -isc^N & s^2c^{N-1} & s^2c^{N-2} & \dots & s^2 \end{pmatrix} .$$

Note that both the density matrices $\rho_A^N(T)$ and $\rho_B^N(T)$ have the same aa -component. Correspondingly, we obtain the same formula as (8) (with $a = \uparrow$) for the probability of finding the upward spin-state by the final detector D_0 at time T . This is the answer to the question asked at the end of the preceding section. That is, we cannot conclude that observation of $P^N(T)$ going to unity is an experimental evidence in support of the naive WFC.

4 The uncertainty relation and other situations

Undoubtedly, one of the most important quantities is

$$\phi \equiv \omega t = \frac{\mu B l}{\hbar v} (= \frac{\pi}{2N}), \quad \text{in } P_{\uparrow}^N(T) = \left[1 - \left(\frac{\mu B l}{\hbar v} \right)^2 \right]^N \quad (12)$$

where l stands for the length of each magnet and v for the neutron speed.

Mathematically, ϕ is of order $O(N^{-1})$, but we cannot take the infinite N limit for the following reasons: (i) In practice, we cannot make the zero limit of the magnetic region, and (ii) in principle, it is impossible to avoid uncertainties Δv and Δx , because

$$\phi \sim \phi_0 \equiv \frac{\mu B l}{\hbar v_0} > \frac{\mu B \Delta x}{\hbar v_0} > \frac{\mu B}{2m v_0 \Delta v} = \frac{1}{4} \frac{\Delta E_m}{\Delta E_k} \quad (13)$$

where v_0 , $\Delta E_m = 2\mu B$ and $\Delta E_k = \Delta(mv^2/2)$ are the mean neutron speed, the magnetic energy gap and the neutron kinetic energy spread, respectively. Consequently, we should have

$$P_{\uparrow}^N(T) \simeq \left[1 - \frac{1}{32} \left(\frac{\Delta E_m}{\Delta E_k} \right)^2 \right]^N \quad (14)$$

For this reason we can set the following limitation:

$$N_{\max} \sim 10^4 .$$

Additionally, we have to take into account the probability of neutron leakage or absorption, $\sigma < 1$, at each step, which should modify the probability of finding the neutron as follows:

$$\tilde{P}_{\uparrow}^N(T) = \sigma^N P_{\uparrow}^N(T) . \quad (15)$$

We cannot take the limit $N \rightarrow \infty$ also for this reason, but we can estimate this kind of loss factor, both experimentally or theoretically, in order to get the net effect.

5 Concluding remarks

We have shown that observation of the QZE does not signify any experimental evidence of the naive WFC (the simple projection), and found the reason why Itano et al's experiment got the same result as Cook's one, even though they did not exactly follow Cook's proposal. We have also examined an important limitation arising from the uncertainty relations and other limitations to be imposed on observation of QZE.

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